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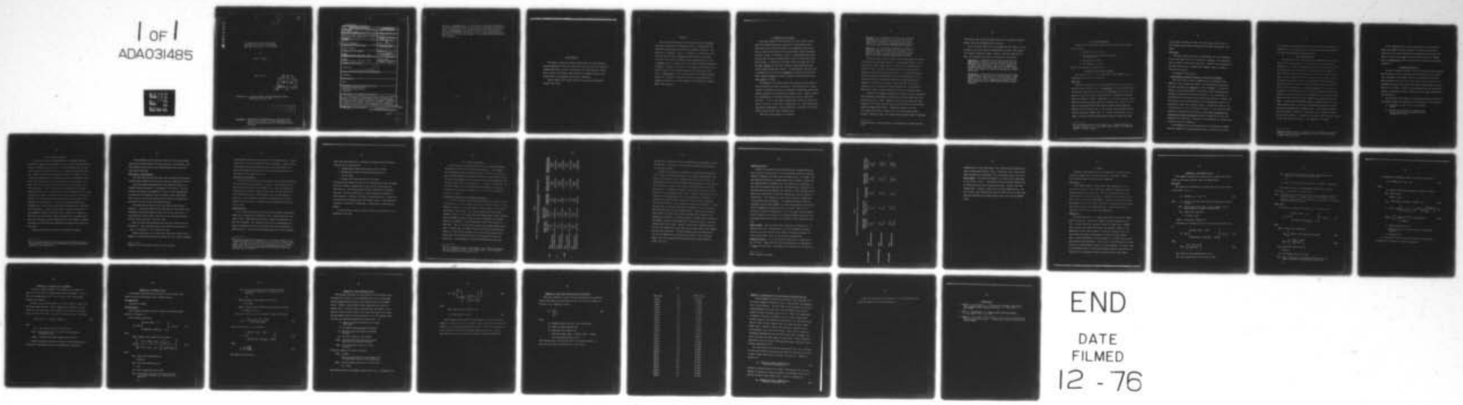

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An Examination of the United States Air Force (Q,R) Policies for Managing Depot-Base Inventories: A Pilot Study

by

Leroy B. Schwarz

May 28, 1976

Prepared for: Air Force Business Research Management Center, Wright-Patterson AFB, Ohio

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The current United States Air Force (Q,R) policy for managing depot-base inventories of nonreparable spares is examined and compared with three alternative policies via computer simulation. Two questions are examined: ~~First, is it~~ (1) whether it is possible to formulate an alternative policy which has the same desirable properties of current Air Force policy (ease of computation and implementation)

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and (2) whether it is

but which is somewhat closer to optimal from a cost and/or backorder performance standpoint? ~~Second, is it~~ possible to formulate an alternative policy which has most of the desirable properties of current policy, but which is substantially closer to optimal from a cost and/or backorder performance standpoint? Affirmative answers to both questions are suggested, based on a very small number of items examined for approximately 1200 item-years.

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Special thanks to Mr. Victor Presutti of the Air Force Logistics Command Study Office.

Abstract

The current United States Air Force (Q,R) policy for managing depot-base inventories of nonreparable spares is examined and compared with three alternative policies via computer simulation. Two questions are examined: First, is it possible to formulate an alternative policy which has the same desirable properties of current Air Force policy (ease of computation and implementation) but which is somewhat closer to optimal from a cost and/or backorder performance standpoint? Second, is it possible to formulate an alternative policy which has most of the desirable properties of current policy, but which is substantially closer to optimal from a cost and/or backorder performance standpoint? Affirmative answers to both questions are suggested, based on a very small number of items examined for approximately 1200 item-years.

I. Objective of the Study

This study examines the United States Air Force (USAF) current policy for managing depot-base inventories of nonreparable spares. This policy may be loosely described as an independent (Q,R) policy; that is, each stocking point in the system (depot or base) determines how much to order, Q, and when to order, R, using models which ignore the fact that the given stocking point is part of a depot-base distribution system. In particular, the model used to determine (Q,R) for the depot assumes that the depot supplies customer demand directly, without going through base inventory. Correspondingly, the model used to determine (Q,R) for the bases assumes that each base reviews stock from a supplier outside the system, instead of receiving stock from a depot within the system.

The designers of the USAF current (Q,R) policy were well aware of these assumptions and their consequence: the current policy is intentionally a suboptimal one from the standpoint of minimizing system order, holding, acquisition, and backorder-day penalty costs. However, despite this suboptimality, the current USAF policy has two very desirable qualities: (1) it is very easily computed (Q's and R's are easily computed); and (2) it is very easily implemented; i.e., each stocking point may operate independently, without bothering to coordinate its policies and/or operations with other stocking points.

This pilot study examines two questions:

Question 1: Is it possible to formulate an alternative inventory policy for depot-base systems which has the same desirable properties of the current USAF policy (cited above), but which is somewhat closer to optimal from a cost and/or backorder performance standpoint?

Question 2: Is it possible to formulate an alternative policy for depot-base systems which has most of the desirable properties of the current USAF policy (cited above), but which is substantially closer to optimal from a cost and/or backorder performance standpoint?

In order to answer these questions we have constructed three alternative depot-base inventory policies: a system myopic policy, an allocation policy, and a depot service. These policies are described in Section II below. We have also designed and constructed a computer simulation of a depot-base inventory system. This simulation program, which is described in Section III below, has the capability of simulating the day-by-day transaction operations for any item currently managed by the USAF (Q,R) inventory system according to any given inventory policy, including the current USAF policy. The simulation model places orders, receives goods, fills orders, accumulates costs.

In Section IV we report the results of a study in which we simulated and compared the performance of USAF current policy and the three alternative policies on 3 items. Demand and cost data for these items were supplied by Mr. Victor Presutti and Ms. Gloria Picciano of the Air Force Logistics Command Study Office. Performance measures used were: (1) average annual order + holding cost; (2) average annual order + holding + acquisition cost; (3) average annual backorder-days[1] accumulated

[1] A backorder-day is the equivalent of one backorder outstanding for one day.

at the bases; and (4) total average annual costs, including estimated penalty costs for backorder-days at the bases.

Due to the small number of items examined for this report, the conclusions of the study most be correspondingly tentative, particularly with respect to Question 1 above. However, with this reservation in mind, the following conclusions are suggested by the results:

> Conclusion 1: Within the set of policies which have the same desirable properties of easy computability and easy implementation as current USAF policy, there appears to be enough system performance sensitivity to suggest that a better policy than current USAF policy exists. However, further investigation is required to identify and confirm this policy.

> Conclusion 2: Within the set of policies which have most of the desirable properties of current USAF policy, there appears to be at least one policy, the allocation policy mentioned above, which is substantially closer to optimal than current USAF policy from a cost and/or performance standpoint.

II. Policies Examined

In this section we briefly describe each of the policies examined in this study:

1. Current Air Force Policy
2. The "System Myopic" Variant to Current Air Force Policy
3. The Allocation Policy
4. The "Depot Only" Policy

Details are provided in Appendices A-D.

Current Air Force Policy (CURRENT)

The current Air Force policy (henceforth called CURRENT) uses an independent (Q,R) policy for each item.

Base (Q,R)

Each base stocking the given item determines how much to order, Q, and an inventory reorder level, R, independent of the costs and/or policy of the depot supplying the item to it and independent of the costs and policies of any other bases stocking the item. Whenever the inventory position (on-hand + on-order - backorders), I, of a given item at a given base falls below its reorder point, R, a quantity Q is ordered which is equal to an adjusted[2] Wilson EOQ (Economic Order Quantity) plus the deficit, if any, between R and I. This restores the inventory position for the item to the adjusted EOQ + R. The base reorder point R is set equal to the given item's average demand during the nominal lead time

[2] If Wilson EOQ exceeds one year's supply, it is adjusted downwards to one year's supply; if it is less than 30 days' supply, it is adjusted upwards to 30 days' supply.

for the depot to supply the item to the base plus a safety stock, S, as a cushion against above average demand or longer than nominal lead times.

Depot (Q,R)

The depot stocking the given item determines its (Q,R) independent of the costs and/or policies of the bases it supplies. The Q determination is similar to that used by the bases. However, the determination of the depot safety stock is somewhat more complex, based on a model by Presutti and Trepp [1].

See Appendix A for details.

A System Myopic Variant of Current Air Force Policy (MYOPIC)

The system myopic variant of CURRENT policy (henceforth called MYOPIC) is also a simple (Q,R) policy. The reorder levels prescribed for bases and depots are identical to those of CURRENT. The only difference is in the determination of the depot and base order quantities. Instead of being based on the Wilson EOQ model, the Q calculations in the MYOPIC policy are based on the system myopic heuristic of Schwarz [3]. This heuristic has been demonstrated to yield near-optimal Q values for deterministic depot-base inventory systems. Although this model is only a deterministic one, as is the Wilson EOQ model, it does provide a simple scheme for determining base and depot Q values which incorporates the interactions of costs and policies between the depot and its bases and among the bases themselves. The details for determining the system myopic Q values are given in Appendix C.

If the MYOPIC policy can be demonstrated to be superior to CURRENT from the standpoint of annual operating and/or acquisition costs, then

we will have found a policy which has all of the desired properties of CURRENT policy, with approximately the same backorder-day performance,[3] and lower operating and/or acquisition costs.

The Allocation Policy

The allocation policy (henceforth called ALLOCATION) uses the same depot and base order quantities and the same reorder levels as the CURRENT policy. However, whenever depot on-hand inventory falls below its reorder level, the depot enters a rationing mode, and remains in this mode until depot on-hand inventory rises above the reorder level again. Any base demand received by the depot when the depot is in the rationing mode initiates a rationing calculation which is designed to fill that demand only partially, reserving some inventory in anticipation of other base demands. The quantity shipped to the given base in response to its demand depends upon the quantity demanded by that base and the anticipated quantities to be demanded by all of the other bases before the depot receives its next shipment (and leaves the rationing mode). In this manner ALLOCATION attempts to spread the risk of a customer backorder across all bases, and consequently improve the overall backorder performance of the CURRENT policy. Any discrepancy between the order quantity demanded by a given base and the quantity shipped by the depot remains as a backorder at the depot to be filled upon receipt of the next depot order quantity. See Appendix D for details.

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[3]Since the MYOPIC R values and safety stocks are identical to those of CURRENT, the backorder-day performance of these two policies should be approximately the same.

If the ALLOCATION policy can be demonstrated to be substantially better than CURRENT from the standpoint of annual operating and/or acquisition costs and/or backorder-day performance, then we shall have found a policy which has most of the desirable properties of CURRENT which is substantially closer to optimal from a cost and/or performance standpoint.

The Depot Only Policy

In order to obtain an estimate of the best possible annual operating and/or acquisition cost of the depot-base inventory system, the "depot only" (henceforth called DEPOT) policy was formulated and evaluated. Under the DEPOT policy all of the bases and base level inventories are eliminated from the system and all base level customer demand is served directly by the depot. The only costs accumulated are the depot order, holding, and acquisition costs. Backorder-days for depot shortages are also accumulated.

The anticipated improvements in annual operating and/or acquisition cost between CURRENT and ALLOCATION are attributable to the following:

(1) Elimination of base level cycle and safety stocks;

(2) Reduction in variability of demand as seen by the depot and the consequent reduction in depot safety stock.

III. Method of Analysis

In order to evaluate the CURRENT, MYOPIC, ALLOCATION, and DEPOT policies, a computer simulation model of a one-depot-N-base distribution system was designed and constructed. For each simulated day, for each item, base level demand for each of up to 31 bases is read from a data set. Demands are filled if sufficient inventory is on hand; any deficiencies are backordered and the backorder-days are accumulated; reorder levels are checked, orders placed according to the given policy, and order costs are accumulated; holding costs for on-hand inventory are also accumulated. Similar calculations are performed for the depot, except backorders are not accumulated.[4] Partial shipments from the depot to the base are made if inventory and/or policy dictates. Deliveries to the bases are made in accordance with the lead (order and ship) times specified for the given base. Deliveries to the depot are made in accordance with the lead time specified for the given item. Lead times are deterministic. Every effort was made to carefully replicate the most important elements of the real system. However, for simplicity and clarity we have excluded returns and quantitatives from our model. Also, for convenience, we have modelled depot sales demands as demands occurring at a fictitious base operating at the depot with a lead time of one day.

Details of the simulation model are available on request.

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[4]This is because backorders at the depot are only significant to the extent that they lead to base level backorders, which are accumulated in the base level calculations.

In the remainder of this section we describe the cost and demand data used in the simulation, the initialization of the simulation, and the procedure used to collect cost and performance data for each of the policies evaluated.

Data Input to the Simulation

Cost and demand data for each item, base, and depot were generated from raw data provided by the Air Force Logistics Command Study Office.[5]

Base level demand was generated for the simulation using a stationary Poisson distribution. See Appendix E for details. Ten years of daily base demands were generated for each item for each data set. Ten data sets were generated using 10 different random number seeds. Thus a total of 100 years of simulated daily demand was generated for each item to be used in the tests reported below.

Lead times used between the depot and each base are listed in Appendix E. Lead times between the outside supplier and the depot were contained in the data provided for each item, as were unit costs. In cases where more than one depot lead time or unit cost were provided, the most recent lead time or cost were used.

Depot and base order processing costs were set at $270.16 and $5, respectively. Depot and base annual policy holding costs were set at .2 and .5, respectively, of the item's unit cost.

Average requisition size for the depot safety stock calculation in CURRENT was calculated from the data provided (the most recent 16 quarters

[5] Thanks to Mr. Victor Presutti and Ms. Gloria Picciano.

of unit demand divided by the total number of orders received). Average requisition size was not updated during the simulation. Initial values of mean depot demand, mean absolute deviation of depot demand, and mean daily demand for each base were determined from a two-year "pre-simulation" of the depot-base system.

Simulation Initialization

For each of the ten data sets used and for each of the four policies tested, the system was initialized as follows: depot inventories were set at one-half the depot Q (from CURRENT) plus lead time demand plus an additional one month's supply for each item; base inventories were set at one-half the base Q (from CURRENT) plus lead time demand for each item. Initial pipeline inventories (outstanding depot and base orders) were set to zero. The system was then simulated for a period of six years.[6]

Data Collection

At the end of the initialization period, the counters accumulating order costs, holding costs, acquisition costs, and backorder-days were reset to zero. Depot and base inventories, pipeline inventories, etc., were left unchanged. The system was then run for an additional four years (16 quarters) and costs and backorder-days were accumulated on a quarterly basis. The total costs and backorder-days for four simulated

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[6]Six years was empirically determined to be the minimum number of years required for each of the tested policies to stabilize, or reach "steady state." Any data collected for a given policy prior to its reaching steady stage would have reflected the transitional properties of the given policy, not the long run properties of the policy, which are the focus of our interest.

years were then divided by 4, yielding the following vector of observations for the 10 year data set:

1. Average Annual Order and Holding Cost for all Items;
2. Average Annual Order, Holding and Acquisition Cost for all Items;
3. Average Annual Backorder-Days for all Items.

This process was repeated for each of the tested policies using the <u>same</u> data set of demand, yielding four vectors (one for each policy) for each data set. This entire process was then repeated for 9 additional data sets. The entire process for 3 items involved the simulation of 1200 item-years = 3 items x 4 policies x 10 years/data set x 10 data sets. The result of the simulation runs was 120 data items = 3 data items/data set/policy x 4 policies x 10 data sets. Detailed data is available on request.

In the following Section we report the results of statistical tests performed on this data.

IV. Analysis of Results

In this section we present the results of a statistical analysis performed on the data collected from the simulation. In so doing, we are attempting to answer two general questions. The first question is: what cost and backorder performance would the Air Force observe if each of the policies examined in the simulation were actually implemented for the items tested? The second question is: is there a statistically significant difference in cost and/or backorder performance between the alternative policies and the CURRENT policy, or did these differences occur only by chance?

Estimates of Annual Cost and Backorder Performance

In order to estimate the cost and backorder-day performance of each of the policies, we have computed the mean and standard deviation of the performance measures of interest along with 95% confidence limits using the Student t distribution.[7] The results appear in the first three columns of Table 1. For example, the mean average annual order + holding cost for CURRENT over the ten data sets was \$4,109/year; the standard deviation was \$60/year. The 95% confidence limits are \$4,170/year to \$4254/year. That is, we may be 95% certain that the <u>true</u> average annual order + holding costs for CURRENT is between \$4170/year and \$4254/year. Similarly, we may be 95% confident that the true average annual order + holding cost for the ALLOCATION policy lies between \$3,213/year and \$3,297/year. Correspondingly, we may be 95% confident that the true

[7] The 95% confidence interval using Student t with 9 degrees of freedom is: $m \pm 2.262s/\sqrt{10}$, where m = the observed mean; s = the observed standard deviation; and sample size = 10.

Table 1

Means, Standard Deviations, and 95% Confidence Limits on Costs and Backorder-Day Performance for the Policies Examined

		Average Annual Order + Holding Cost ($)	Average Annual Order + Holding + Acquisition Cost ($)	Average Annual Backorder-Days	Average Annual Total Cost Using $12.24 ($)	Average Annual Total Cost Using $32.64 ($)
RENT	Mean	$4,109	$17,550	456	23,127	48,384
	Standard Deviation	60	783	143	1,366	1,537
	95% Confidence Limits	$4,066-4,152	$16,990-18,110	354-558	22,150-24,104	47,284-49,484
PIC	Mean	4,212	17,655	372	22,207	29,793
	Standard Deviation	58	586	182	2,269	5,928
	95% Confidence Limits	4,170-4,254	17,236-18,074	242-502	20,584-23,830	25,552-34,034
CATION	Mean	3,255	16,696	18	16,912	17;273
	Standard Deviation	61	730	8	749	805
	95% Confidence Limits	3,213-3,297	16,174-17,218	12-24	16,376-17,448	16,697-17,849
T	Mean	1,831	15,831	997	28,058	48,384
	Standard Deviation	90	781	478	5,639	15,370
	95% Confidence Limits	1,767-1,895	15,272-16,390	655-1339	24,024-32,092	37,388-59,380

average annual backorder-days for the MYOPIC policy lies between 242/year and 502/year. Differences in the observed performance of the policies will be analyzed below.

In order to facilitate the examination of differences between policies we have constructed an estimate of the marginal backorder-day cost; that is, an estimate of what a backorder-day costs the Air Force. By multiplying the average annual backorder-day performance of a given policy by this estimate we obtain the estimated annual backorder cost of the given policy. By adding the estimated annual backorder cost of a given policy to its corresponding annual order + holding + acquisition cost, an estimate of the <u>annual total</u> cost of the given policy is obtained. We estimated the marginal backorder-day cost using a procedure suggested by Mr. Victor Presutti. This procedure is described in Appendix F. Using this procedure we estimated the marginal backorder cost to lie between \$12.24/backorder-day and \$32.64/backorder-day. Since the range between these numbers was fairly large, \$20.40, we used both estimates in all subsequent calculations. The results appear in columns 4 and 5 of Table 1. For example, using \$12.24/backorder-day in column 4 of Table 1, the mean average annual total (order + holding + acquisition + backorder penalty) cost for the CURRENT over the ten data sets was \$23,127/year; the standard deviation was \$1,366/year. The 95% confidence limits on average total cost for the CURRENT policy are \$22,150 to \$24,104. Similar average annual total costs using the \$32.64/backorder-day estimate appear in column 5 of Table 1.

Significance Tests

In Table 2 we present paired mean differences, standard deviations, and t-statistics between each of the three alternative policies examined (MYOPIC, ALLOCATION, and DEPOT) and the CURRENT policy. For example, the mean observed difference in average annual order + holding + acquisition cost between MYOPIC and CURRENT policy was $105/year; the difference in mean observed backorder-days/year was -84 (MYOPIC had on average 84 fewer backorder-days/year than CURRENT; the difference in mean observed average total costs using the $12.24 ($32.64) backorder-day cost was -$921/year (-$2631/year). The t-statistics indicate the statistical significance of the observed mean differences. In a simple comparison of a given alternative policy versus CURRENT, a t-value whose absolute value exceeds 1.833 is regarded as significant at the .05 level.[8] That is, if we observed a t-statistic greater than 1.833 or less than -1.833, we reject the null hypothesis that there is no difference between the policies and be at least 95% confident that a real difference exists between the policies.

MYOPIC-CURRENT: The only statistically significant difference lies in average annual order + holding costs; all other differences observed (an average 84 unit decrease in the number of backorder-days/year and lower average annual costs) are statistically insignificant.

ALLOCATION-CURRENT: All of the observed differences are significant at the .05 level. Indeed all of the observed differences are significant at above the .0005 level. This means that on the basis of our tests the

[8] With 9 degrees of freedom.

Table 2

Mean Paired Differences Between Alternative and Current Policies

		Average Annual Order Holding Cost ($)	Average Annual Order + Holding + Acquisition ($)	Average Annual Backorder-Days	Average Annual Total Cost Using $12.24 ($)	Average Annual Total Cost Using $32.64 ($)
MYOPIC-CURRENT	Mean	103	105	-84	-921	-2631
	Standard Deviation	58	329	188	2,284	6098
	t	5.62	1.01	1.41	-1.27	-1.36
ALLOCATION-CURRENT	Mean	-854	-854	-438	-6215	-15,151
	Standard Deviation	65	65	144	1716	4,655
	t	-41.67	-41.59	-9.61	-11.46	-10.29
DEPOT-CURRENT	Mean	-2,278	-1719	542	4,911	15,960
	Standard Deviation	110	1471	526	6,009	16,633
	t	-65.56	-3.69	3.26	2.58	3.03

ALLOCATION policy has significantly lower average annual holding costs, lower average annual holding + order + acquisition costs, lower average annual backorder-day performance, and lower average annual total costs. DEPOT-CURRENT: All observed differences are statistically significant at the .05 and .01 level. This means that we may be 99% confident that the DEPOT policy has lower average annual order + holding, and lower average order + holding + acquisition costs than CURRENT policy. However, we may also be 99% confident that the DEPOT policy has worse backorder-day performance and higher average total costs than the CURRENT policy.

V. Summary

The results and conclusions of this report must be tentative ones because the sample size of items tested (3) is very small. However, the following tentative conclusions are suggested.

Conclusion 1:

If the MYOPIC policy is indeed better than CURRENT policy, our tests fail to show it. No statistically significant differences were observed between the two policies. This may be accounted for by the characteristics of the items studied. Alternatively, the MYOPIC policy may, in fact, be no better than CURRENT. However, the detailed cost and performance data for the CURRENT, ALLOCATION, and MYOPIC policies suggest that a better policy than CURRENT may exist. Further examination is required to identify and confirm this hypothesis.

Conclusion 2:

The ALLOCATION policy is a significantly better policy than CURRENT policy in every way. There are a number of possible explanations for this. The most intuitively plausible reason is simply that in CURRENT policy safety stocks and reorder points are improperly modeled. [The DEPOT model assumes that the depot is supplying customers in unit lot sizes rather than bases with minimum lot sizes of 30 days' demand.] There is some corroboration of this hypothesis in the DEPOT policy's very poor backorder-day performance. Recall that in the DEPOT policy the depot serves all customer demands directly. If this hypothesis is correct, then the ALLOCATION policy's outstanding performance might be explained by its rationing of the (inadequate) depot reorder and safety stock supply.

Appendix A: The CURRENT Policy

This Appendix describes the calculations of a given item's (Q,R) values for the bases and depot under the CURRENT policy.

Base (Q,R):

The formula for determining the reorder level for a given item at a given base j, R_j, is:

$$R_j = \text{Int}[DDR_j \times L_j + SS_j + .5] \quad \text{(A1)}$$

where L_j = nominal lead (order and ship) time from depot to base j, in days;

DDR_j = daily demand rate at base j (total demand at base j during the last 365 days divided by 365);

SS_j = safety stock for base j

$= [3 \times DDR_j \times L_j]^{\frac{1}{2}}$;

INT[X] = the greatest integer $\leq$ X;

The formula for determining the Q for a given item at base j, Q_j, is:

$$Q_j = \text{Max} \left\{ \begin{array}{l} \text{Int}[30 \times DDR_j + .999]; \\ \\ \text{Int}[\text{Min}\{365 \times DDR_j, EOQ_j\} + .999] \end{array} \right\} + (R_j - I_j) \quad \text{(A2)}$$

where

$$EOQ_j = \left[\frac{2 \times 365 \times OC_j}{HC_j + UC} \right]^{\frac{1}{2}} \quad \text{(A3)}$$

OC_j = base j's order processing cost = \$5;

UC = unit acquisition cost of the given item;

HC_j = cost to hold each unit of the given item/year at base j, expressed as a fraction of its unit cost

= .2;

I_j = base j's inventory position (on-hand + on-order - backorders);

The values of Q_j and R_j are recomputed whenever base inventory position, I_j, falls below the reorder level.

Note: The safety stock calculation is based on the assumption that base lead time demand is a compound Poisson with mean $DDR_j \times L_j$ and variance to mean ratio of 3. Hence SS_j is set equal to one standard deviation of lead time demand.

Depot (Q,R):

The formula for determining depot Q, Q_D, for a given item is:

$$Q_D = \text{Max}\left\{\begin{matrix}\text{Int}[6 \times MDR_D + .5] \\ \\ \text{Int}[\text{Min}\{36 \times MDR_D; EOQ_D\} + .5]\end{matrix}\right\} + (R_D - I_D) \qquad \text{(A4)}$$

where

MDR_D = monthly depot demand rate

$$= \frac{1}{24} \sum_{n=1}^{8} [\text{Demand in } n^{th} \text{ most recent quarter}] \qquad \text{(A5)}$$

$$EOQ_D = \left[\frac{2 \times MDR_D \times OC_D}{HC_D \times UC}\right]^{1/2} \qquad \text{(A6)}$$

OC_D = depot order processing cost

= $270.61;

UC = unit acquisition cost of item;

HC_D = cost to hold each unit of the given item/year at the depot, expressed as a fraction of its unit cost

= .5.

The formula for determining depot R, R_D, for a given item is:

$$R_D = \mathrm{Int}[MDR_D \times L_D + SS_D + .5] \quad \text{(A7)}$$

where

L_D = nominal lead time from supplier to depot, in months;

$$SS_D = \text{safety stock}$$

$$= \mathrm{Max}\{K \times \sigma_D; 0\} \; ;$$

$$\sigma_D = .5945 \times MAD_Q \times (0.82375 + 0.42625 \times L_D) \quad \text{(A8)}$$

$$K = 0.707 \times \log_e \left[\frac{\lambda}{2 \times HC_D \times UC \times R^{\frac{1}{2}}} \cdot \frac{\sigma \times (1 - e^{-\sqrt{2}\, EOQ/\sigma})}{\sqrt{2} \times EOQ_D} \right] \quad \text{(A9)}$$

$$MAD_Q = \frac{1}{8} \sum_{n=1}^{8} [\text{Demand in } n^{th} \text{ most recent quarter} - (3 \times MDR_D)] \quad \text{(A10)}$$

R = average requisition size

= demand during last 24 months divided by the number of orders;

λ = shortage factor.

The derivation of SS_D may be found in Presutti and Trepp [1]. A procedure for estimating λ is given in Appendix B.

Appendix B: Estimation of λ in CURRENT

In order to simulate CURRENT it was necessary to estimate the value of λ to be used in the depot safety stock calculation. See (A9). Upon the recommendation of Mr. Victor Presutti, AFLC, the following procedure was employed.

The basic idea of the procedure is to find a value of λ which, for the chosen sample of items, yields a total dollar investment in depot safety stock approximating a depot safety stock investment of 53 days' supply for each item. That is, we wish to find a value of λ satisfying

$$\sum_i UC_i \times SS_i(\lambda) = 53 \sum_i UC_i \times MDR_{Di}/30 \qquad \text{(B1)}$$

where

UC_i = unit acquisition cost of the i^{th} item;

$SS_i(\lambda)$ = depot safety stock for the i^{th} item determined by equation (A9);

MDR_{Di} = average monthly depot demand rate for item i.

A computer program was written to find the λ value satisfying (B1). The value of λ which came closest to satisfying (B1) was $113.25.

Appendix C: The MYOPIC Policy

This Appendix describes the calculation of a given item's (Q,R) values for the bases and depot under the MYOPIC policy.

R Determination:

Identical to CURRENT

Q Determination:

The Q values for depots and bases is based on the system myopic heuristic of Schwarz [3].

For the depot Q_D satisfies

$$Q_D = \text{Max} \left\{ \begin{array}{l} \text{Int}[6 \times MDR_D + .5] \\ \\ \text{Int}[\text{Min}\{36 \times MDR_D; Q_D'\} + .5] \end{array} \right\} + (R_D - I_D) \qquad \text{(C1)}$$

where

MDR_D = monthly depot demand rate (see (A5));

$$Q_D' = \left[\frac{2 \times 12 \times MDR_D \times (OC_D + OC_B \sum_{j=1}^{N} n_j)}{UC \times (HC_D + (HC_B - HC_D) \times \sum_{j=1}^{N} (n_j MDR_j / MDR_D))} \right]^{1/2} \qquad \text{(C2)}$$

where

OC_D = depot order processing cost

= \$270.16;

OC_B = base order processing cost

= \$5;

UC = unit acquisition cost of item;

HC_D = cost to hold each unit of the given item/year at the depot, expressed as a fraction of its unit cost

= .2;

HC_B = cost to hold each unit of the given item/year at each base, expressed as a fraction of its unit cost;

= .5;

MDR_j = average monthly demand rate at base j

= $30 \times DDR_j$;

MDR_D = average annual demand rate at the depot (see (A5));

N = number of bases;

The n_j value for base j is the smallest integer n satisfying

$$n(n+1) \geq \frac{OC_D \times (HC_B - HC_D) \times MDR_j}{OC_B \times HC_D \times MDR_D} \tag{C3}$$

The Q value for base j, Q_j, satisfies:

$$Q_j = \text{Max} \begin{Bmatrix} \text{Int}[30 \times DDR_j + .999] \; ; \\ \\ \text{Int}[\text{Min}\{365 \times DDR_j ; Q_j'\} + .999] \end{Bmatrix} \tag{C4}$$

where

$$Q_j' = \frac{Q_D \times MDR_j}{n_j \times MDR_D} \tag{C5}$$

See Schwarz [3] for details.

Appendix D: The ALLOCATION Policy

This Appendix describes the calculations of the ALLOCATION policy. As described in Section II, the ALLOCATION policy uses the same depot and base order quantities and the same reorder levels as the CURRENT policy. However, whenever depot on-hand inventory falls below its reorder level, the depot enters a rationing mode, and remains in this mode until depot on-hand inventory rises above the reorder level again. Any base demand received by the depot when it is in the rationing mode initiates the rationing calculation described below. Let

k = index of the base initiating the rationing calculation;

N = the number of bases served by the depot;

Q_j = the most recent order quantity demanded by base j;

D_j = the date of base j's last demand;

$DATE_D$ = the anticipated date when the next shipment will be received by the depot;

DDR_j = the average daily demand rate served by base j;

From these quantities the depot calculates

$$CYCL_j = Q_j/DDR_j$$

= the anticipated number of days between base j's last order and base j's next order; and

$DATE_j$ = the anticipated date of base j's next order

$$= D_j + CYCL_j$$

From these quantities the shipment quantity base k, S_k, is computed to be:

$$S_k = \mathrm{Min} \left\{ \begin{array}{l} Q_k \; ; \\ \mathrm{Int} \left[OH_D \times \dfrac{W_k \times Q_k}{\sum\limits_{j=1}^{N} W_k \times Q_k} \right] \end{array} \right\} \qquad \text{(D1)}$$

where

OH_D = depot on-hand inventory; and

$$W_j = \mathrm{Max}\{0; DATE_D - DATE_j\} \qquad \text{(D2)}$$

The discrepancy between base k's order quantity, Q_k, and the amount shipped by the depot to base k, S_k; that is, $[Q_k - S_k]$, will be automatically shipped to base k when depot inventories permit. In our cost calculations, it is assumed that these extra shipments incur no extra cost. However, such additional costs may be easily included in subsequent evaluations.

Appendix E: Base Level Demand Generation Procedure

Base level demand for a given item was generated for the simulation using a stationary Poisson distribution with the mean daily demand rate for base j, D_j, computed as follows:

$$D_j = \frac{F_j}{\sum_{i=1}^{N} F_i} \cdot \tag{E1}$$

where

F_j = Weighting factor for base j (see table below);

N = Number of bases handling item;

M = Mean depot daily demand rate

= Historical depot sales + transfer demand - returns for 16 quarters divided by 1440.

The following table lists the base code; base weighting factor, F_j, and base lead times for the bases used in this study.

Base Code	F_j	Lead Time, L_j
FB2647	0.3	19 days
FB2823	3.0	11 days
FB4801	9.6	9 days
FB4802	9.6	9 days
FB4803	7.6	12 days
FB4809	7.2	10 days
FB4812	9.7	11 days
FB4829	6.6	10 days
FB4852	3.7	9 days
FB4857	3.6	12 days
FB4814	8.9	13 days
FB4887	0.6	8 days
FB5000	2.4	17 days
FB5210	5.4	28 days
FB5219	4.8	28 days
FB5250	3.0	25 days
FB5264	1.8	25 days
FB5270	9.0	16 days
FB5284	3.6	17 days
FB5294	1.8	20 days
FB5529	3.6	22 days
FB5573	7.2	18 days
FB5587	3.6	32 days
FB5606	6.6	22 days
FB5612	3.6	22 days
FB5620	4.2	18 days
FB5621	4.8	18 days
FB5643	5.4	19 days
FB5644	4.8	19 days
FB5688	1.8	14 days

Appendix F: Estimation of Air Force Marginal Backorder-Day Cost

In this Appendix we describe the procedure used to estimate the Air Force's marginal backorder-day cost. This procedure was suggested by Mr. Victor Presutti. The basic idea of the procedure is as follows: in setting depot safety stock at 53 days of supply versus 52 days' supply or 54 days of supply, the Air Force is implicitly stating that it prefers to incur the incremental cost of 53 days' supply versus 52 days' supply to the incremental backorder-day cost that would result from the lower (52 days' supply) versus the higher (53 days' supply) safety stock. Similarly, the Air Force implicitly prefers to save the incremental cost of 53 versus 54 days' safety stock to saving the incremental backorder-day cost that would result from the higher (54) versus the lower (53) days' supply of safety stock. These two implicit preferences allow us to place lower and upper bounds on the Air Force's marginal backorder-day cost.

The lower bound on the marginal backorder-day cost, LB, is obtained by running the simulation program using CURRENT policy with 52 and then 53 days of depot safety stock (using the same data set). Then LB is computed as:

$$LB = \frac{\text{Increase in Order + Holding Costs}}{\text{Decrease in Backorder-Days}} \tag{F1}$$

Similarly, the upper bound on the marginal backorder-day cost, UB, was obtained by running the simulation program using CURRENT policy with 53 and then 54 days of depot safety stock. Then UB is computed as:

$$UB = \frac{\text{Increase in Order + Holding Costs}}{\text{Decrease in Backorder Days}} \tag{F2}$$

Using these procedures LB was computed to be \$12.24/backorder-day and UB was computed to be \$32.64/backorder-day.

Bibliography

1. Presutti, V. and Trepp, R. C., "More Ado about Economic Order Quantities," Naval Research Logistics Quarterly, 17:2 (June, 1970); pp. 243-251.

2. Neter, J. and Wasserman, W., Applied Linear Statistical Models, p. 146, Richard Irwin, Inc., 1974.

3. Schwarz, L. B., "Single Cycle Continuous Review Policies for Arborescent Production/Inventory Systems." Prepared under contract F33615-75C-5277 for the AFBRMC.